MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2022

COMPUTER BASED TEST (CBT)

DATE : 26-06-2022 (MORNING SHIFT) | TIME : (9.00 AM to 12.00 PM)

Duration 3 Hours | Max. Marks : 300

QUESTIONS & SOLUTIONS

PART : PHYSICS

1. An expression for dimensionless quantity P is given by $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$; where α and β are constants, x is distance; k is Boltzmann constant and t is the temperature. Then the dimensions of α will be :

(A) $[M^0 L^{-1} T^0]$
(B) $[M L^0 T^{-2}]$
(C) $[M L T^{-2}]$
(D) $[M L^2 T^{-2}]$

Ans. **(C)**

Sol. $\left[\frac{kt}{\beta.x}\right] = \left[M^0L^0T^0\right]; \left[\frac{ML^2T^{-2}}{\beta \times L}\right] = \left[M^0L^0T^0\right]$

$[\beta] = [M.L.T^{-2}]$

But s P is dimensionless

$\therefore [\alpha] = [\beta] = [M.L.T^{-2}]$

2. A person is standing in an elevator. In which situation, he experience weight loss?

(A) When the elevator moves upward with constant acceleration
(B) When the elevator moves downward with constant acceleration
(C) When the elevator moves upward with uniform velocity
(D) When the elevator moves downward with uniform velocity

Ans. **(B)**

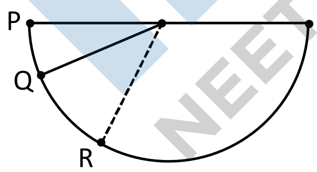
Sol. $W_{app} = m.g_{eff}$

3. An object is thrown vertically upwards. At is maximum height, which of the following quantity becomes zero?

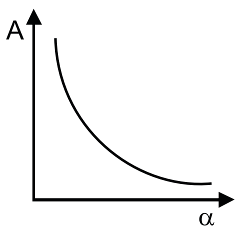
(A) Momentum
(B) Potential Energy
(C) Acceleration
(D) Force

Ans. **(A)**

4 A ball is released from rest from point P of a smooth semi-spherical vessel as shown in figure. The ratio of the centripetal force and normal reaction on the ball at point Q is A while angular position fo point Q is α with respect to point P. Which of the following graphs represent the correct relation between A and α when ball goes Q to R ?



(A)



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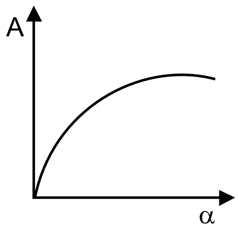

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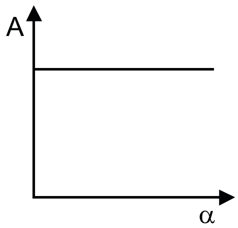

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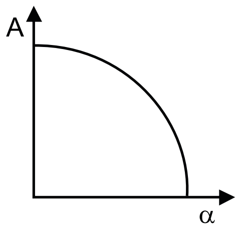


Ans. **(C)**

Sol. $V=\sqrt{2gh}=\sqrt{2gR\sin\theta}, Fc=\frac{mv^2}{R}=2mg\sin\theta$

$N=mg\sin\theta+\frac{mv^2}{R}=3mg\sin\theta, \frac{F_c}{N}=\frac{2}{3}=\text{const.}$

5. A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2 rads^{-1} in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in rads^{-1})

(A) $\frac{M}{(M+m)}$

(B) $\frac{(M+2m)}{2m}$

(C) $\frac{2m}{(M+2m)}$

(D) $\frac{2(M+2m)}{M}$

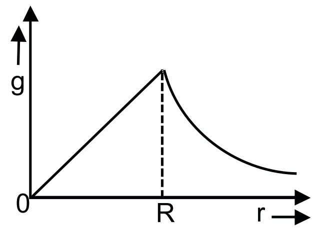
Ans. **(C)**

Sol. Conservation of angular momentum gives

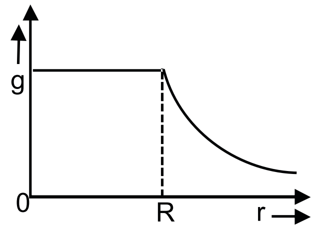
$\Rightarrow MR^2\ \omega1 = (m+2m)\ R^2\ \omega2$

$\therefore\ \ \omega2 = \left(\frac{M}{M+2m}\right)\omega_1$

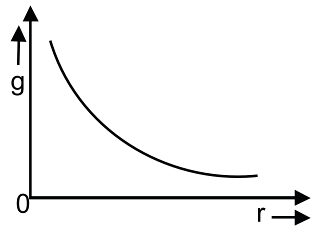
6. The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by:
(Given R = radius of earth)
(A)



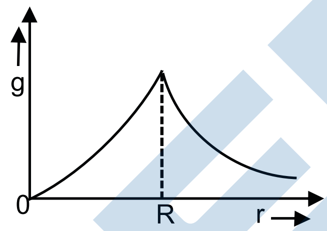
(B)



(C)



(D)



Ans. **(A)**

Sol. $g = \left(\frac{GM}{R^3}\right).r$ for r < R

$\sqrt{\frac{5}{6}}T$ for r > R

7. The efficiency of a Carnot's engine, working between steam pint and ice point, will be :
(A) 26.81%
(B) 37.81%
(C) 47.81%
(D) 57.81%

Ans. **(A)**

Sol. $\eta = 1 - \frac{T_{less}}{T_{more}} = 1 - \frac{273}{373} = 26.8\%$

8. Time period of a simple pendulum in a stationary lift is 'T'. If the lift accelerates with $\frac{g}{6}$ vertically upwards then the time period will be:
(Where g = acceleration due to gravity)
(A) $\sqrt{\frac{6}{5}}T$
(B) $\sqrt{\frac{5}{6}}T$

(C) $\Delta T = \frac{M_0V^2}{5R}$

(D) $\sqrt{\frac{7}{6}}T$

Ans. **(C)**

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$

$T^1 = 2\pi\sqrt{\frac{L}{g+\frac{g}{6}}} = 2\pi\sqrt{\frac{6L}{g}}$

$\frac{T^1}{T} = \sqrt{\frac{6}{7}}$

9. A thermally insulated vessel contains an ideal gas of molecular mas M and ratio of specific heats 1.4. Vessel is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surrounding and vessel temperature of the gas increases by:
(R=universal gas constant)

(A) $\frac{Mv^2}{7R}$

(B) $\frac{Mv^2}{5R}$

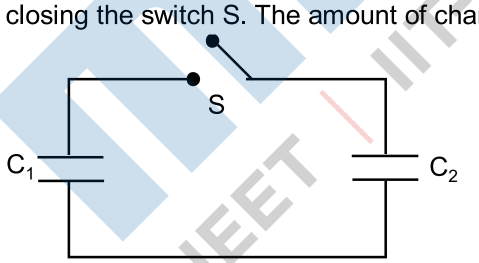
(C) $2\frac{Mv^2}{5R}$

(D) $7\frac{Mv^2}{5R}$

Ans. **(B)**

Sol. $\frac{1}{2}MV^2 = n\left(\frac{f}{2}R\right)\Delta T; \frac{1}{2}(nM_0)v^2 = n\frac{5}{2}R\Delta T\ ;\ \Delta T = \frac{M_0V^2}{5R}$

10. Two Capacitors having capacitance C_1 and C_2 respectively are connected as shown in figure. Initially, capacitor C_1 is charged to a potential difference V volt by a battery. The battery is then removed and the charged capacitor C_1 is now connected to uncharged capacitor C_2 by closing the switch S. The amount of charge on the capacitor C_2 after equilibrium, is :



(A) $\frac{C_1C_2}{(C_1+C_2)}v$

(B) $\frac{(C_1+C_2)}{C_1C_2}v$

(C) $(C_1+C_2)V$

(D) $(C_1-C_2)V$

Ans. **(A)**

Sol. Common potential $V' = \frac{V_1C_1}{(C_1+C_2)}$

So, change on capacitor C_2

$Q_z = C_2 V' = C_2 \left(\frac{V_1C_1}{C_1 + C_2}\right)$

11. Given below two statements : One is labeled as Assertion (A) and other is labelled as Reason (R).

Assertion (A): Non polar materials do not have any permanent dipole moment.

Reason (R): When a non-polar material is placed in an electric field, the centre of the positive charge distribution of it's individual atom or molecule coincides with the centre of the negative charge distribution.

In the light of above statements, choose the most appropriate answer from the options given below.

(A) Both **(A)** and **(R)** are correct and (R) is the correct explanation of (A).
(B) Both **(A)** and **(R)** are correct and **(R)** is not the correct explanation of **(A)**
(C) **(A)** is correct but **(R)** is not correct.
(D) **(A)** is not correct but **(R)** is correct.

Ans. **(A)**

Sol. $|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}[5t^2 + 4t^2 + 2t - 5] = 15t^2 + 8t + 2$

at t = 2s $|\varepsilon| = 15 \times 2^2 + 8 \times 2 + 2 = 78v$

$\therefore I = \frac{|\varepsilon|}{R} = \frac{78}{5} = 15.6A$

13. An aluminium wire is stretched to make its length, 0.4% larger. The percentage change in resistance is:

(A) 0.4%
(B) 0.2%
(C) 0.8%
(D) 0.6%

Ans. **(C)**

Sol. $R = \rho\frac{L}{A}$ but volume LA = constant so $A \propto \frac{L}{A}$

so $R \propto L^2$

$\frac{\Delta R}{R} \times 100 = 2\left(\frac{\Delta L}{L} \times 100\right) = 2 \times (0.4) = 0.8\%$

14. A proton and an alpha particle of the same velocity enter in a uniform magnetic field which is acting perpendicular to their direction of motion. The ratio of the radii of the circular paths described by the alpha particle and proton is :

(A)1:4
(B) 4:1
(C) 2:1
(D) 1:2

Ans. **(C)**

Sol. For circular path in magnetic field

$r = \frac{mV}{qB}$

So,

	α	p
m	4	1
Q	2e	e

$r_1 : r_2 = \frac{4}{2e} : \frac{1}{e} = \frac{2}{1} = 2:1$

15. If Electric field intensity of a uniform plane electro magnetic wave is given as

$E = -301.6 \sin(kz-\omega)\ \hat{a}x + 452.4 \sin(kz-\omega t)\ \hat{a}y \frac{V}{m}$.

Then, magnetic intensity 'H' of this wave in Am^{-1}, will be:

[Given : Speed of light in vacuum $c = 3 \times^{10} ms^{-1}$, Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$]

(A) $+0.8\sin(kz-\omega t)\ \hat{a}y + 0.8\sin(kz-\omega t)\ \hat{a}x$.

(B) $+1.0\times10^{-6}\sin(kz-\omega t)\ \hat{a}y + 1.5\times10^{-6}(kz-\omega t)\ \hat{a}x$

(C) $-0.8\sin(kz-\omega t)\ \hat{a}y - 1.2\sin(kz-\omega t)\ \hat{a}x$

(D) $-1.0\times10^{-6}\sin(kz-\omega t)\ \hat{a}x - 1.5\times10^{-6}\sin(kz-\omega t)\ \hat{a}x$

Ans. **(C)**

Sol. B is perpendicular to $\vec{E}$ and

$B_0 = \frac{E_0}{C}$

Also $H = \frac{B}{\mu}$

16. In free space, an electromagnetic wave of 3 GHz frequency strikes over the edge of an object of size $\frac{\lambda}{100}$, where λ is the wavelength of the wave in free space. The phenomenon, which happens there will be:
(A) Reflection
(B) Refraction
(C) Diffraction
(D) Scattering

Ans. **(D)**

17. An electron with speed v and a photon with speed c have the same de-Broglie wavelength. If the kinetic energy and momentum of electron are E_e and p_e and that of photon are E_{ph} and p_{ph} respectively. Which of the following is correct ?

(A) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$

(B) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

(C) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$

(D) $\frac{p_e}{p_{ph}} = \frac{v}{2c}$

Ans. **(B)**

Sol. For both

$\lambda = \frac{h}{P} \Rightarrow P = \frac{h}{\lambda}$

So P will be same for e & photon

$KE_e = \frac{1}{2}mv^2 = \frac{PV}{2}$

$KE_{ph} = mC^2 = PC$

$\frac{KE}{KE_{ph}} = \frac{V}{2C}$

18. How many alpha and beta particles are emitted when Uranium $_{92}U^{238}$ decays to lead $_{82}Pb^{206}$?
(A) 3 alpha particles and 5 beta particles
(B) 6 alpha particles and 4 beta particles
(C) 4 alpha particles and 5 beta particles
(D) 8 alpha particles and 6 beta particles

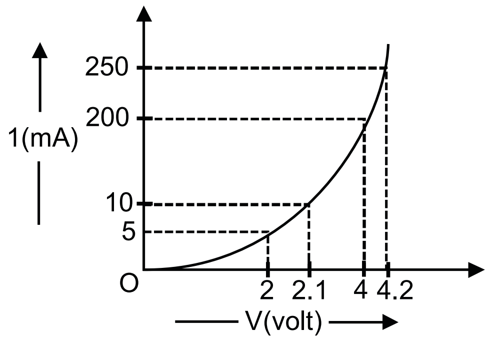
Ans. **(D)**

Sol. $_{92}U^{238} \longrightarrow {}_{82}Pb^{206} + n_1({}_2\alpha^4) + n_2({}_{-1}\beta^0)$

$238 = 206 + 4n_1 + n_1(0) \Rightarrow n_1 = 8$

$92 = 82 + (n_1)(2) + (2) + (n_2)(-1) \Rightarrow n_2 = 6$

19. The I-V characteristics of a p-n junction diode in forward bias is shown in the figure. Ratio of dynamic resistance, corresponding to forward bias voltage of 2V and 4V respective is :



(A) 1:2
(B) 5:1
(C) 1:40
(D) 20:1

Ans. **(B)**

Sol. $R = \frac{\Delta V}{\Delta i}$

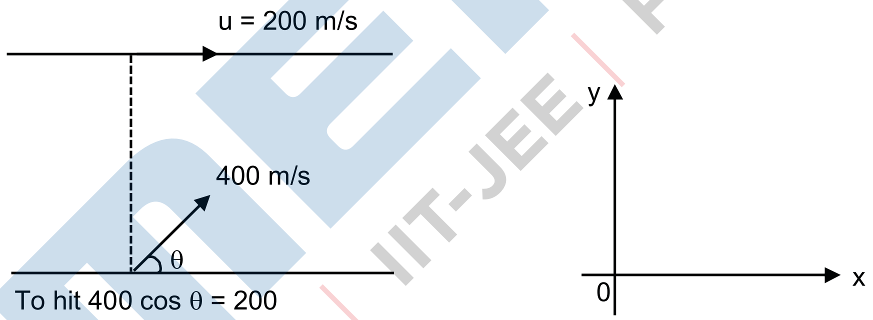
20. Choose the correct statement for amplitude modulation :
(A) Amplitude of modulating signal is varied in accordance with the information signal.
(B) Amplitude of modulated signal is varied in accordance with the information signal.
(C) Amplitude of carrier signal is varied in accordance with the information signal.
(D) Amplitude of modulated signal is varied in accordance with the modulating signal.

Ans. **(C)**

21. A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1}. When it passes directly overhead an anti-aircraft gun, a bullet is fried from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be_________°.

Ans. **60.00**

Sol.



To hit 400 cos θ = 200

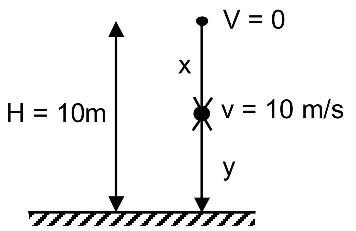
{∴ Both travel equal distance along horizontal, of their start and coordinates an x axis are same}

$\Rightarrow$ $\theta = 60°$ **Ans.**

22. A ball of mass 0.5 kg is dropped from the height of 10 m. The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is _________m. [Use g = 10 m/s^2]

Ans. **5.00**

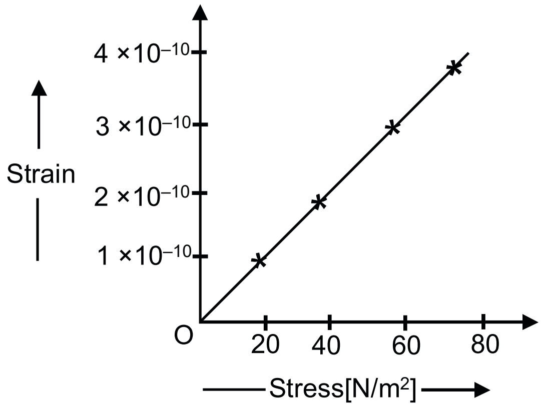
Sol.



From equation of motion from top
$V^2 = u^2 + 2as$; $10^2 = 0 + 2gx$
X = 5 m

So, height from ground y = H – x = 10 5 = 5 m

23. The elastic behavior of material for liner stress and linear strain, is shown in the figure. The energy density for a linear strain of 5×10^{-4} is ___________kJ/m³. Assume that material is elastic upto the linear of 5×10^{-4}.



Ans. 25.00

Sol. From given graph

$$Y = \frac{\text{stress}}{\text{strain}}$$

Energy density = $\frac{1}{2}$Y(strain)²]

24 The elongation of a wire on the surface of the earth is 10^{-4} m. The same wire of same dimensions is elongated by 6×10^{-5} m on another planet. (Take acceleration due to gravity on the planet will be _________ms^{-2})

Ans. 06.00

Sol $$Y = \frac{\frac{mg+0}{2}}{\frac{YA}{\ell_0}} = \frac{mg\ell_0}{2YA}$$

$$X \propto g \Rightarrow \frac{g_2}{g_1} = \frac{x_2}{x_1} \Rightarrow \frac{g_2}{10} = \frac{6\times10^{-5}}{10\times10^{-5}}$$

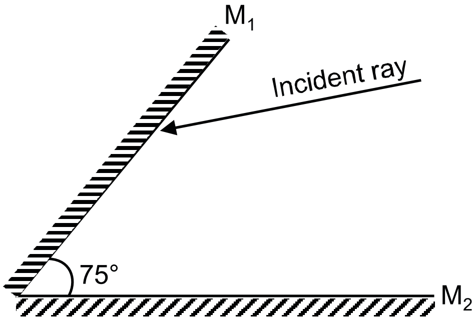
$g_2 = 6$

25. A 10 Ω, 20 mH coil carrying constant current is connected to a battery of 20 V through a switch. Now after switch is opened current becomes zero in 100 μs. The average e.m.f. induced in the coil is___________ V.

Ans. 400

Sol. $$\varepsilon = \frac{\Delta\phi}{\Delta t} = \frac{L\Delta i}{\Delta t}$$

26. A light ray is incident, at an incident angle θ_v on the system of two plane mirrors M_1 and M_2 having an inclination angle 75° between them (as shown in figure). After reflecting from mirror M_1 it gets reflected back by the mirror M_2 with an angle of reflection 30°. The total deviation of the ray will be __________ degree.



Ans. **210**

Sol. $d = 360° - 2\theta = 360° - 150° = 210°$

27. In a vernier calipers, each cm on the main scale is divided into 20 equal parts. If tenth vernier scale division coincides with nineth main scale division. Then the value of vernier content will be ___________ $\times 10^{-2}$ mm.

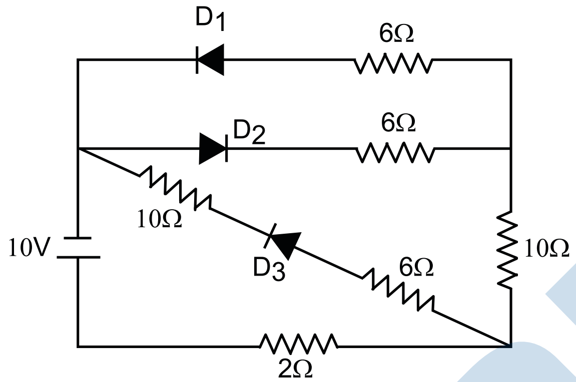
Ans. **0.500**

Sol. V.C. = 1 MSD – 1VSD

$$= 1\text{ MSD} - \frac{9}{10}\text{ MSD}$$

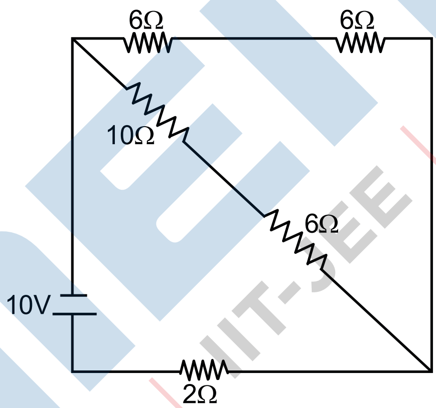
$$= \frac{1}{10}\text{ MSD} = \frac{1}{10} \times \frac{1\text{cm}}{20} = \frac{1}{200}\text{ cm} = \frac{1000}{200} \times 10^{-2}\text{mm}$$

28. As per the given circuit, the value of current through the battery will be __________ A.



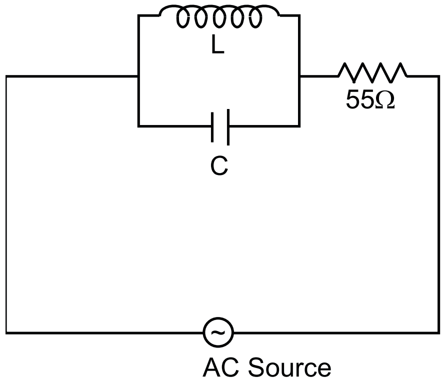
Ans. **01.00**

Sol. Effective CKt is



$$i = \frac{10}{10} = 1\text{A}$$

29. A 110 V, 50 Hz, AC source is connected in the circuit (as shown in figure). The current thrugh the resistance 55 Ω, at resonance in the circuit, will be _________ A.



Ans. **00.00**

Sol $X_L = i(L\omega), X_c = -\frac{i}{C\omega}$

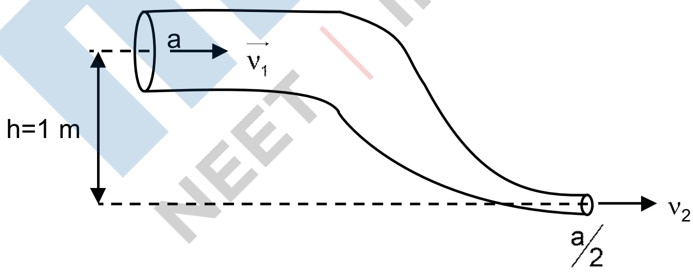
Their resultant will be $\frac{X_C X_L}{X_C + X_L} = \frac{(i(L\omega))\left(-\frac{i}{C\omega}\right)}{iL\omega + \left(-\frac{i}{C\omega}\right)}$

$= \frac{\frac{L}{C}}{\left(L\omega - \frac{i}{C\omega}\right)} = -\frac{L}{C\left(L\omega - \frac{i}{C\omega}\right)}$

In case of resonance $\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow L\omega = \frac{1}{C\omega} \Rightarrow |Z| \to \infty \Rightarrow I \to 0$

30. An ideal fluid of density 800 kgm^{-3}, flows smoothly through a bent pipe (as shown a in figure) that tapers in cross-sectional area from a to $\frac{a}{2}$, The pressure difference between the wide and narrow sections of pipe is 4100 Pa. At wider section, the velocity of fluid is $\frac{\sqrt{x}}{6}$ ms^{-1} for x= __________. (Given g = 10 ms^{-2})



Ans. **363**

Sol. From following eq[n]

$\frac{1}{2}\rho V_A^2 + \rho gh + P_A = \frac{1}{2}\rho V_B^2 + \rho gh + Pa,$

$V_B = 2\ V_A$ and

$P_A - P_A = 4100$ Pa.

PART : CHEMISTRY

1. a commercially sold conc. HCl is 35% HCl by mass. If the density of this commercial acid is 1.46 g/ml, the molarity of this solution is :
(A tonic mass: Cl = 35.5 amu, H = 1 amu)
(A) 10.2 M
(B)12.5 M
(C)14.0 M
(D)18.2 M

Ans **(C)**

Sol. $M = \frac{(w/w\%)d.10}{(MM)_{Solute}}$

$M = \frac{35 \times 1.46 \times 10}{36.5} = 14.0M$

2. An evacuated glass vessel weighs 40.0 g when empty, 135.0 g when filled with a liquid of density 0.95 g mL^{-1} and 40.5 when filled with an ideal gas at 0.82 atm at 250 k. The molar mass of the gas in g mol^{-1} is:
(Given : R = 0.082 L atm K^{-1} mol^{-1})
(A) 35
(B) 50
(C) 75
(D) 125

Ans. **(D)**

Sol. Weight of gas = 40.5 – 40 = 0.5g
Mass of liquid = 135 – 40 = 95g.

Volume of glass vessel = $\frac{95.0g}{0.95g/ml}$
= 100 ml.

$\therefore$ PV = $\left(\frac{W}{M}\right)$ RT

$(0.82\text{ at})(100\times10^{-3}) = \left(\frac{0.5}{M}\right)(0.0821)(250)$

M = 125 g/mol

3. If the radius of the 3rd Bohr's orbit of hydrogen atom is r_3 and the radius of 4th Bohr's is r_4. Then :

(A) $r_4 = \frac{9}{16}r_3$

(B) $r_4 = \frac{16}{9}r_3$

(C) $r_4 = \frac{3}{4}r_3$

(D) $r_4 = \frac{4}{3}r_3$

Ans. **(B)**

Sol. $r_{n,z} = 0.529\frac{n^2}{z}$Å

$r_{3,H} = 0.529 \times \frac{9}{1}$Å

$r_{4,H} = 0.529 \times \frac{16}{1}$Å

$\therefore \frac{r^3}{r^4} = \frac{9}{16}$

$r_4 = \frac{16}{9}r_3$

4. Consider the tons/ molecule

$O_2^+, O_2, O_2^-, O_2^{2-}$
(A) $O_2^{2-} < O_2^- < O_2 < O_2^+$
(B) $O_2^-, O_2^{2-}, O_2, O_2^+$
(C) $O_2^-, O_2^{2-}, O_2^+, O_2$
(D) $O_2^-, O_2^+, O_2^{2-}, O_2$

Ans. **(A)**

Sol.

Species	O_2^{+2}	O_2^+	O_2	O_2^-	O_2^{-2}
No. of e–	14	15	16	17	18
Bond order	3	2.5	2	1.5	1

5. The $\left(\frac{\partial E}{\partial T}\right)_P$ of different types of half cells are as follows:

A	B	C	D
1×10^{-4}	2×10^{-4}	0.1×10^{-4}	0.2×10^{-4}

(Where E is the electronic force)
Which of the above half cells would be preferred to be used as reference electrode.
(A) A
(B) B
(C) C
(D) D

Ans. **(C)**

Sol. Metal which have lower value of $\left(\frac{dE}{DT}\right)_P$ is used for standard or reference half electrode.

6. Choose the correct stability order of group 13 elements in their +1 oxidation state.
(A) Al < Ga < In < Tl
(B) Tl < In < Ga < Al
(C) Al < Ga < Tl < In
(D) Al < Tl < Ga < In

Ans. **(A)**

Sol. In boron family as we move down the group. stability of +1 state increase due to inert pair effect.

7. Given below are two statements:
Statement I : According to the Ellingham diagram, any oxide with higher $\Delta G°$.
Statement II : The metal involved in the formation of oxide placed lower in the Ellingham diagram can reduce the oxide of metal placed higher in the diagram.
In the light of the above statements, choose the **most appropriate** answer from the options given below:
(A) Both **Statement I** and **Statement II** are correct.
(B) Both **Statement I** and **Statement II** are incorrect.
(C) **Statement I** is correct but **Statement II** is incorrect.
(D) **Statement I** is incorrect but **Statement II** is correct.

Ans. **(D)**

Sol. → Higher value of ΔG like HgO, Ag_2O are less stable.
→ In Ellingham diagram lower situated metals is more reactive, it can reduce higher metal oxide.

8. Consider the following reaction:

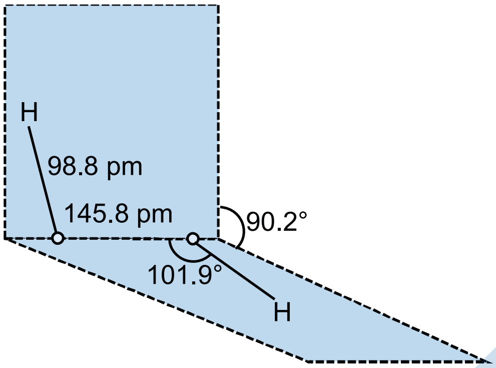
$$2HSO_4^-(aq) \xrightarrow[(2)\text{Hydrolysis}]{(1)\text{Electrolysis}} 2HSO_4^- + 2H^+ + A$$

The dihedral angle in product A in its solid phase at 110 K is:
(A) 104°
(B) 111.5°
(C) 90.2°
(D) 111.0°

Ans. **(C)**

Sol. Peroxodisulphate, obtained by electrolytic oxidation of acidified sulphate solutions at high current density, on hydrolysis yields hydrogen peroxide.

$$2HSO_4^-(aq) \xrightarrow[(2)Hydrolysis]{(1)Electrolysis} HO_3SOOSO_3H(aq) \xrightarrow{Hydrolysis} 2HSO_4-(aq) + 2H^+(aq) + H_2O_2(aq)$$



Solid phase

9. The correct order of melting points is:
(A) Be > Mg > Ca > Sr
(B) Sr > Ca > Mg > Be
(C) Be > Ca > Mg > Sr
(D) Be > ca > Sr > Mg

Ans. (D)

Sol.

Property	Beryllium Be	Magnesium Mg	Calcium Ca	Strontium Sr	Barium Ba	Radium Ra
m.p./ K	1560	924	1124	1062	1002	973

10. The correct order of melting points of hydrides of group 16 elements is:
(A) $H_2S < H_2 S_e < H_2Te < H_2O$
(B) $H_2o < H_2S < H_2Se < H_2Te$
(C) $H_2S < H_2Te < H_2Se < H_2O$
(D) $H_2Se < H_2S < H_2Te < H_2$

Ans. (A)

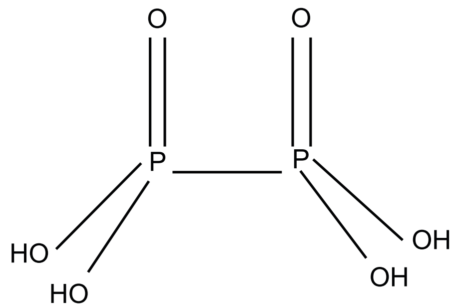
Sol. In general molar mass ↑ ,MP ↑ but in H_2O due to H–bonding, it is maximum among the following:

Property	H_2O	H_2S	H_2Se	H_2Te
m.p. / K	273	188	208	222

11. Consider the following rection :]
A + alkali → B (Major Product)
If B is an oxoacid of phosphorus with no P-H bond, then A is:
(A) White P_4
(B) Red P_4
(C) P_2O_3
(D) H_3PO_3

Ans. (B)

Sol. Red P on reaction with alkali gives pyrophosphoric acid ($H_4 P_2 O_6$)



12. Polar stratospheric clouds facilitate the formation of :
(A) $CLONO_2$
(B) HOCl
(C) ClO
(D) CH_4

Ans. **NTA answer is (B) Zigyan answer is (C)**

Sol. Polar stratospheric clouds (PSCs) surfaces act as catalysts that convent more forms benign forms of human-made chlorine into active free radicals (for example monoxide). During the return of spring sunlight these radicals destroy many ozone molecules in a series of chain reactions.

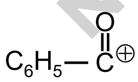
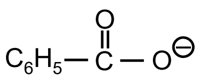
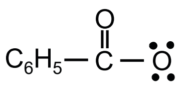
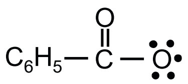
13. Given below are two statements:
Statement I: In 'lassaigne's Test', when both nitrogen and sulphur are present in an organic compound, sodium thiocyanate is formed.
Statement II If both nitrogen and sulphur are present in an organic compound, then the excess of sodium used in sodium fusion will decompose the sodium thiocyanate formed to give NaCN and Na_4S.
In the light of the above statements, choose the most appropriate answer from the options given below:
(A) Both **statement I** and **Statement II** are correct.
(B) Both **Statement I** and **Statement II** are incorrect.
(C) **Statement I** is correct but **Statement II** is incorrect.
(D) **Statement I** is incorrect but **statement II** is correct.

Ans. **(A)**

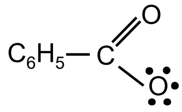
Sol. Both statement – I & II are correct.
(In case, nitrogen and sulphur both are present in an organic compound, then sodium thicoyana red color) is formed with neutral $FeCl_3$

$$Na + C + N + S \longrightarrow NaSCN$$
$$\text{Neutral } FeCl_3 + NaSCN \longrightarrow Fe(SCN)_3)$$
Blood red

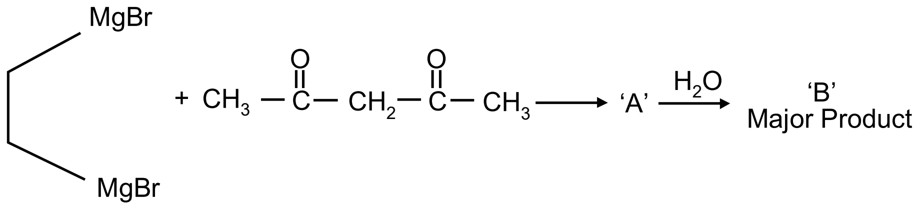
If Na is taken in excess, it destroy SCN^- and form Na_2S and NaCN.

14. $(C_7H_5O_2)_2 \xrightarrow{h\nu} [x] \rightarrow 2C_6H_5 + 2CO_2$
Consider the above reaction and identify the intermediate 'X'
(A)

(B)

(C)

(D)


Ans. **(D)**

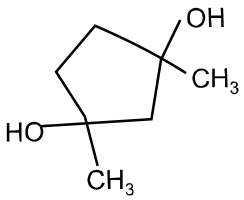
Sol. Benzoate radical  is formed as intermediate.

15.

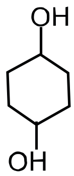


Consider the above reaction sequence and identify the product B.

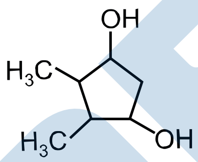
(A)



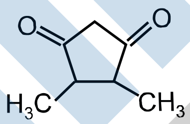
(B)



(C)

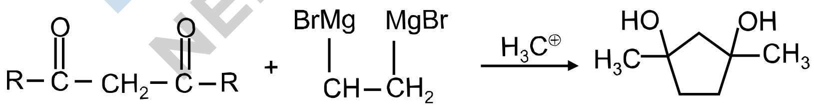


(D)



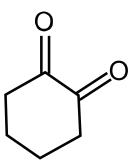
Ans. **(B)**

Sol.

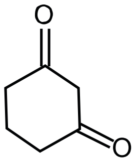


16. Which will have the highest enol content?

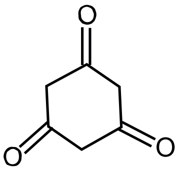
(A)



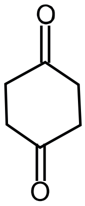
(B)



(C)

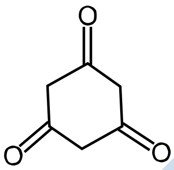
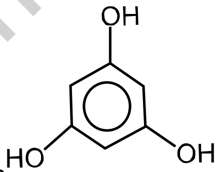


(D)



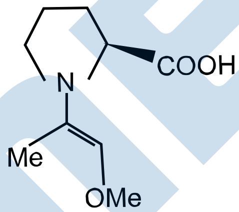
Ans. **(C)**

Sol.

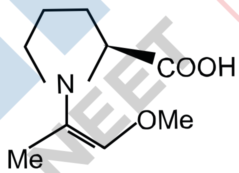
Enol form of (4)  is more stable due to formation of aromatic  benzene- 1,3,5-triol.

17. Among the following structures, which show the most enamine formation? (Where Me is – CH_3)

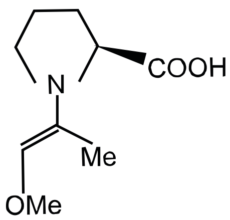
(A)



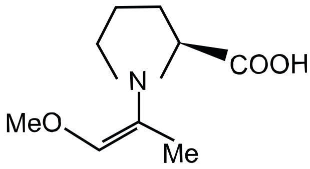
(B)



(C)



(D)



Ans. **NTA answer is (C), Zigyan answer is (B)**

Sol. Enamine form is more stable due to intramolecular hydrogen bonding.

18. Which of the following sets are correct regarding polymer.

(A) Copolymer: Buna-S
(B) Condensation polymer: Nylon-6,6
(C) Fibers : Nylon-6,6
(D) Thermosetting polymer: Terylene
(E) Homopolymer: Buna-N

Choose the correct answer from given options below:

(A) (A), (B) and (C) are correct
(B) (B), (C) and (D) are correct
(C) (A), (C) and (E) are correct
(D) (A), (B) and (D) are correct

Ans. **(A)**

Sol. It is fact.

19. A chemical which stimulates the secretion of pepsin is:

(A) Anti histamine
(B) Cimetidine
(C) Histamine
(D) Zantac

Ans. **(C)**

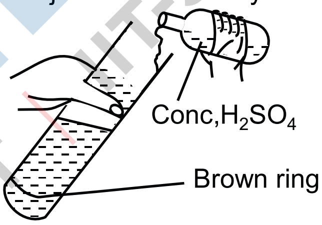
Sol. Excess of acidity (pepsin in stomach) is due to release of excess of histamine. Therefore modern synthetic drugs are antihistamines for the treatment of gastic ulcers by blocking the acid release action of histamine.

20. Which statement is not true with respect to nitrate ion test?

(A) A dark brown ring is formed at the junction of two soulution.
(B) Ring is formed due to nitro ferrous sulphate complex,
(C)The brown complex is $[Fe(H_2O)_5(NO)]SO_4$.
(D) Heating the nitrate salt with conc. H_2So_4, light brown fumes are evolved.

Ans. **(B)**

Sol. Brown ring test: When a freshly prepared saturated solution of iron (II) sulphate is added to nitrate solution and then concentrated H_2SO_4 is added slowly from the side of the test tube, a brown ring is obtained at the junction of two layers.



$NaNO_3 + H_2SO_4 \longrightarrow NaHSO_4 + HNO_3$

$6FeSO_4 + 2HNO_3 + 3H_2SO_4 \longrightarrow 3Fe_2(SO_4)_3 + 2NO + 4H_2O$

$2NO_2^- + 4H_2SO_4 + 6Fe^{2+} \longrightarrow 6Fe^{3+} + 2NO\uparrow + 4SO_4^{2-} + 4H_2O.$

$Fe^2 + NO\uparrow + 5H_2O \longrightarrow$ **$[Fe^I(H_2O)_5NO^+]^{2+}$ (brown ring).**

21. For complete combustion of methanol

$$CH_3OH(I) + \frac{3}{2}O_2(g) \rightarrow CO_2(g) + 2H_2O(1)$$

the amount of heat produced as measured by bomb calorimeter is 726 kJ mol^{-1} at 27°C. The enthalpy of combustion for the reaction is ⁻x $kJmol^{-1}$, where x is ________. (Nearest integer) (Given: R = 8.3 JK^{-1} mol^{-1})

Ans. **(727)**

Sol. $CH_3OH(\ell)$.

For chemical reaction: $\Delta H = \Delta U + \Delta n_{(g)} RT$

$\Delta H = -726 + (-1/2)\, 8.314 \times 10^{-3} \times 300$
$= -727.245$ KJ/Mole
$\therefore x = 727$

22. A 0.5 percent solution of potassium chloride was found to freeze at – 0.24°C. The percentage dissociation of potassium chloride is ________. (Nearest integer)
(Molal depression constant for water is 1.80 K kg mol^{-1} and molar mass of KCl is 74.6g mol^{-1})

Ans. **(98)**

Sol. $\Delta T_f = iK_f m$

$$0.24 = i \times 1.80\left[\frac{0.5 \times 1000}{74.6 \times 99.5}\right]$$

$I = 1.979 = 1 + \alpha;\ \alpha = 0.979 \approx 98\%$

23. 50mL of 0.1 M CH_3COOH is being titrated 0.1 M NaOH, When 25 mL of NaOH has been added, the pH of the solution will be ________$\times 10^{-2}$, (Nearest integer)
(Given: $pK_a(CH_3COOh)=4.76$)
log 2 = 0.30
log 3 = 0.48
log 5 = 0.69
log 7 = 0.84
log 11 = 1.04

Ans. **(476)**

Sol. $CH_3COOH\ (aq) + NaOH\ (aq.) \rightarrow CH_3COONa\ (aq) + H_2O\ (\ell)$

	CH_3COOH	NaOH	CH_3COONa
milli moles	5	2.5	
After reaction Milli moles	2.5	–	2.5

Resultant solution is acidic buffer solution with same concentration of acid and salts. So, pH of Solution pH = pKa = 4.76 = 476 × 10^{-2}.

24. A flask is filed with equal moles of A and B. The half lives of A and B are 100 s and 50 s respectively and are independent of the intial concentration. The time required for the concentration of A to be four times that of B is ________s.

Ans. **(200)**

Sol. For first order reaction

$$k = \frac{0.693}{t_{1/2}}$$

$[A] = 4[B]$
$[A]_0 e^{-kAt} = 4[B]_0\, e^{-kBt}$
$\because$ As $[A]_0 = [B]_0$; $e^{-kBt} = 4e^{-kBt}$
$-kAt = \ln 4 - kBt$
$\therefore$ $t(k_B - k_A) = 2\, \ell n\, 2$

$$t = \frac{2 \times 0.693}{\left(\frac{0.693}{50} - \frac{0.693}{100}\right)} = \frac{2 \times 100}{2-1} = 200\,\text{sec}.$$

25. 2.0 g of H_2 gas is adsorbed on 2.5 g of platinum powder at 300 K and 1 bar pressure. The volume of the gas adsorbed per gram of the adsorbent is __________ ml.
(Given : R= 0.083 L bar K^{-1} mol^{-1})

Ans. **(9960)**

Sol. $$V_{H_2}(g) = \left(\frac{2}{2}\right)\frac{(0.083)300}{1}$$

= 24.9Ld

$\therefore$ Volume of $H_2(g)$ adsorbed on 2.5 Pt = 24.9L

$\therefore$ Volume of $H_2(g)$ adsorbed on 1g Pt $= \frac{24.9}{2.5}$ = 9.96L = 9960 ml

26. The spin-only magnetic moment value of the most basic oxide of vanadium among V_2O_3 V_2O_4 and V_2O_5 is _________ B.M.(Nearest integer)

Ans. **(3)**

Sol. → V_2O_3 (V^{+3}) is most basic oxide amongs the following
→ Total no. of unpaired electrons in v^{+3} is n = 2.

$_{23}V^{+3} = [Ar]3d^2$

$\rightarrow \sqrt{n(n+2)} or \sqrt{8} = 2.87BM \approx 3$

27. The spin-only magnetic moment value of an octahedral complex among $CoCl_3.4NH_3$, $NiCl_2.6H_2O$ and $PtCl_4.2HCl$, which upon reaction excess of $AgNO_3$ gives 2 moles of AgCl is ________ B.M. (Nearest Integer)

Ans. (3)

Sol. (I) $H_2[PtCl_6] \rightarrow$ no ppt with $AgNO_3$

Pt^{+4}, CN = 6}

(II) $[Ni(H_2O)_6]Cl_2 \xrightarrow[\text{excess}]{AgNO_3}$ 2 mole of AgCl ↓,

Ni^{+2}, CN = 6, n = 2, $\mu_m = \sqrt{n(n+2)}BM = \sqrt{8} = 2.82BM \simeq 3$

Aqua complexes, generally have CN = 6

(III) $[CoCl_2(NH_3)_4]Cl \xrightarrow[\text{excess}]{AgNO_3}$ 1 Mole of AgCl ↓

Co^{+3}, CN = 6

28. On complete combustion 0.30 g of an organic compound gave 0.20 g of carbon dioxide and 0.10 g of water. The percentage of carbon in the given organic compound is _________. (Nearest Integer)

Ans. (18)

Sol. Moles of $CO_2 = \frac{0.2}{44}$

Moles of carbon = $\frac{0.2}{44}$

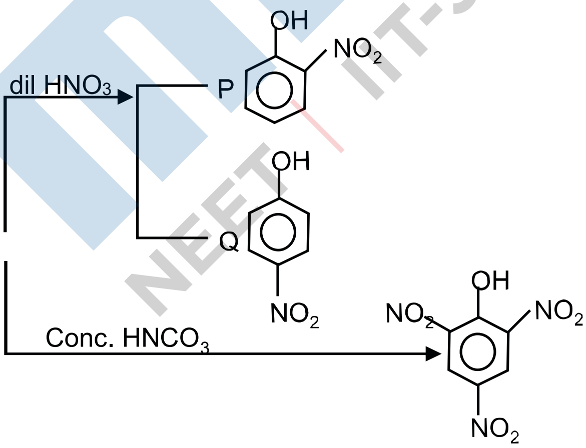
Weight of carbon = $\frac{0.2}{44} \times 12g$

% of carbon = $\frac{0.2}{44} \times 12 \times \frac{100}{0.3} = 18.11$

29. Compound 'P' on nitration with dil. HNO_3 yields two isomers (A) and (B). These isomers can be separated by steam distillation. Isomers (A) and (B) show the intramolecular and intermolecular hydrogen bonding respectively. Compound (P) on reaction with conc. HNO_3 yields a yellow compound 'C' a strong acid. The number of oxygen atoms is present in compound 'C' ________.

Ans. (7)

Sol.

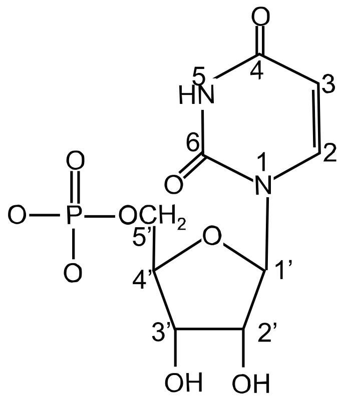


Picric acid

30. The number of oxygens present in a nucleotide formed from a base, that is present only in RNA is __________.

Ans. (9)

Sol.



Nucleotide with uracile base present in RNA.

Maths-Paper
SECTION-A

1. Let $f(x)=\frac{x-1}{x+1}, x \in R-(0,-1,1)$. If $f^{n+1}(x) = f(f^n(x))$ for all $\in N$, then $f^6(6) + f^7(7$ is equal to:

(A) $\frac{7}{6}$ (B) $-\frac{3}{2}$ (C) $\frac{7}{12}$ (D) $-\frac{11}{12}$

Ans. **(B)**

Sol. $f(x)=\frac{x-1}{x+1}$

$\Rightarrow f^2(x)=f(f(x))=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=\frac{1}{x}$

$f^3(x)=f(f^2(x))=f\left(-\frac{1}{1}\right)=\frac{x+1}{x-1}$

$\Rightarrow f^4(x)=f\left(\frac{x+1}{x-1}\right)=\frac{1}{x}$

$\Rightarrow f^6(x)=-\frac{1}{x} \Rightarrow f^6(6)=-\frac{1}{8}$

$f^7(x)=\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$

$\Rightarrow f^7(x)=\frac{8}{-6}=-\frac{4}{3}$

$\therefore -\frac{1}{6}+-\frac{4}{3}=-\frac{3}{2}$

2. Let $\sqrt{2}$]

And $B=\left\{z \in C : \arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$.

Then $A \cap B$ is:

(A) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(B) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies the second quadrant only

(C) an empty set

(D) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Ans. **(B)**

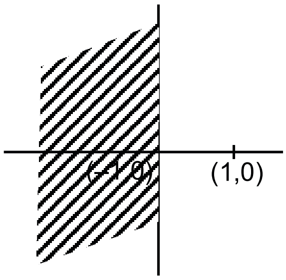
Sol. Set A

$\Rightarrow \left|\frac{z+1}{z-1}\right|<1$

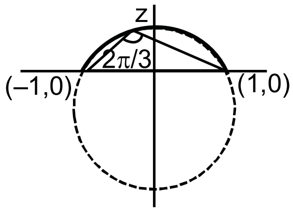
$\Rightarrow |z+1|<|z-1$

$\Rightarrow (x+1)^2 + y < (x-1)^2 + y^2$

$\Rightarrow x < 0$



Set B



centre: $\left(0, -\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{2\pi}{3}$

$\Rightarrow x^2 + y^2 + \frac{2y}{\sqrt{3}} - 1 = 0$

$A \cap B$

$\Rightarrow$ Centre $\left(0, -\frac{1}{\sqrt{3}}\right)$

3. let A be a 3 × 3 invertible matrix. If |adj (24A)| = adj(3adj(2A))|, then $|A|^2$ is equal to:

(A) 6^2 (B) 2^{12} (C) 2^6 (D) 1

Ans. **(C)**

Sol. |adj (24A)| = |adj 3(adj 2A)|

$\Rightarrow |24a|^2 = (3\ \text{adj}(2A))^2$

$\Rightarrow (24^3\ |A|)^2 = (3^2|\text{adj}\ (2A)|)^2$

$\Rightarrow 3^6(|2A|^2)^2$

$\Rightarrow 24^2\ |A|^2 = (24^3|A|)^2 = 3^6 \times 2^{12}\ |A|^4$

$\Rightarrow |A|^2 = \frac{24^6}{3^6 \times 2^{12}} = 64$

4. The ordered pair (a, b), for which the system of linear equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x - y + az = -1$

Has no solutions, is:

(A) $\left(3, \frac{1}{3}\right)$ (B) $\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x)}{1 - \tan(\cos^{-1} x)}$

(C) $\left(-3, -\frac{1}{3}\right)$ (D) $\left(3, -\frac{1}{3}\right)$

Sol. $\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$

3(–8a – 9) + 2(5a – 18) + 1(21) = 0

$\Rightarrow$ a = –3

Also $\begin{vmatrix} 3 & -2 & b \\ 5 & 8 & 3 \\ 2 & 1 & -1 \end{vmatrix}^{\frac{1}{3}}$

If $b = \frac{1}{3}$

$\Delta_2 = 0$ So b must be equal to

$-\frac{1}{3}$

5. The remainder when $(2021)^{2023}$ is divided by 7 is:

(A) 1　(B) 2　(C) 5　(D) 6

Ans. (C)

Sol. $(2021)^{2023} = (7\lambda - 2)^{2023}$

${}^{2023}C_0(7A)^{2023} - \ldots\ldots {}^{2023}C_{2023}2^{2023}$

$= 7t - 2^{2023}$

$\therefore -2^{2023} = -2 \times 2^{2022}$

$= -2 \times (2^3)^{674}$

$= -2(1 + 7\mu)^{674}$

$= -(7\alpha + 2)$

$\Rightarrow$ remainder = –2 or + 5

6. $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1}x)}{1 - \tan(\cos^{-1}x)}$ is equal to:

(A) $\sqrt{2}$　(B) $-\sqrt{2}$　(C) $\frac{1}{\sqrt{2}}$　(D) $-\frac{1}{\sqrt{2}}$

Ans. (D)

Sol. $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1}x)}{1 - \tan(\cos^{-1}x)}$

$\lim\limits_{x \to \frac{1}{\sqrt{2}}} \frac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)}{1 - \tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$

$\lim\limits_{x \to \frac{1}{\sqrt{2}}} \frac{\sqrt{1-x^2} - x}{1 - \left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$

$\lim\limits_{x \to \frac{1}{\sqrt{2}}} (-x) = -\frac{1}{\sqrt{2}}$

7. Let f, g : R $\to$ R be two resl valued functions

Defined as $\begin{cases} -|x+3| & , \quad x < 0 \\ e^x & , \quad x \geq 0 \end{cases}$ and

$g(x)=\begin{cases} x^2+k_1x & , \quad x<0 \\ 4x+k_2 & , \quad x\ge 0 \end{cases}$ where k_1 and k_2 are real constants. If (gof) is differentiable at x = 0, then (gof) (–4) + (gof) (4) is equal to:

(A) $4(e^4 + 1)$ (B) $2(2e^4 + 1)$ (C) $4e^4$ (D) $2(2e^4 – 1)$

Ans. **(D)**

Sol. $f(x)=\begin{cases} x+3 & ; \quad x<-3 \\ -(x+3) & ; \quad -3\le x<0 \\ e^x & ; \quad x\ge 0 \end{cases}$

$g(x)=\begin{cases} x^2+k_1x & ; \quad x<0 \\ 4x+k_2 & ; \quad x\ge 0 \end{cases}$

$\frac{\sqrt{17}+3}{2}$

$g(f(x))=\begin{cases} (x+3)^2+k_1(x+3) & ; \quad x<-3 \\ (x+3)^2-k_1(x+3) & ; \quad -3\le x<0 \\ 4e^x+k_2 & ; \quad x\ge 0 \end{cases}$

check continuity at x = 0

gof (0) = $g(f(0^-)) = g(f(0^+))$

$4 + k_2 = 9 – 3k_1 = 4 + k_2$

$3k_1 + k_2 = 5$ …(a)

differentiate

$g(f(x))'=\begin{cases} 2(x+3)+k_1 & ; \quad x<-3 \\ (x+3)-k_1 & ; \quad -3\le x<0 \\ 4e^x & ; \quad x\ge 0 \end{cases}$

$6 – k_1 = 4$

$k_1 = 2$ ….(b)

$\therefore k_1 = 2, k_1$

$gof(x)=\begin{cases} (x+3)^2+2(x+3) & ; \quad x<-3 \\ (x+3)^2-2(x+3) & ; \quad -3\le x<0 \\ 4e^x-1 & ; \quad x\ge 0 \end{cases}$

gof(–4) + gof(4) = $4e^4 – 2$

$\Rightarrow 2(2e^4 – 1)$

8. The sum of the absolute minimum and the absolute maximum values of the function f(x) = $|3x – x^2 + 2| – x$ in the interval [–1,2] is:

(A) $\frac{\sqrt{17}+3}{2}$ (B) $f\left(-1,\frac{3-\sqrt{17}}{2}\right)$ (C) 5 (D) $\frac{9-\sqrt{17}}{2}$

Ans. **(A)**

Sol. $f(x)=\begin{cases} x^2-4x-2, & \forall x\in\left(-1,\frac{3-\sqrt{17}}{2}\right) \\ -x^2+2x+2, & \forall x\in\left(\frac{3-\sqrt{17}}{2},2\right) \end{cases}$

f'(x) when $x \in f\left(-1, \frac{3-\sqrt{17}}{2}\right)$

f(x) = 2x – 4 = 0 ⇒ x = 2

f(x) = 2(x – 2) ⇒ (x) is always ↓

f(2) = 2

f(–1) = 3

$f\left(\frac{3-\sqrt{17}}{2}\right) = \left(\frac{\sqrt{17}-3}{2}\right)$

f'(x) when $x \in \left(\frac{3-\sqrt{17}}{2}, 2\right)$

f'(x) = 2x + 2

f(x) = –2(x + 1)

f(x) = 0 when x = 1

f(1) = 3

absolute minimum value = $\left(\frac{\sqrt{17}-3}{2}\right)$

absolute maximum value = 3

9. Let S be the of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a,b), $ab \neq 0$, Then:

(A) S = ϕ (B) n(s) = 1 (C) S = {2k : $k \in N$} (D) S = N

Ans. (D)

Sol. $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$

Slope of tangent at (a, b)

$n.\left(\frac{x}{a}\right)^{n-1}.\frac{1}{a}+n\left(\frac{x}{b}\right)^{n-1}.\frac{1}{b}\frac{dy}{dx}=0$

$\left.\frac{dy}{dx}\right|_{(a,b)}=-\frac{b}{a}$

∴ Equation of tangent

$y - b = -\frac{b}{a}(x-a)$

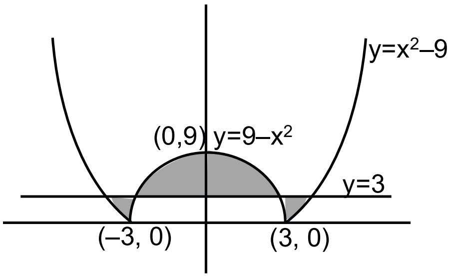
$\frac{x}{a}+\frac{y}{b}=2 \forall n \in N$

10. The area bounded by the curve y = |x^2 – 9| and the line y = 3 is:

(A) $4\left(2\sqrt{3}+\sqrt{6}-4\right)$ (B) $4\left(4\sqrt{3}+\sqrt{6}-4\right)$ (C) $8\left(4\sqrt{3}+3\sqrt{6}-9\right)$ (D) $8\left(4\sqrt{3}+\sqrt{6}-9\right)$

Ans. (DROP)

Sol.



Area of shaded region

$$= 2\int_0^3 \left(\sqrt{9-y} - \sqrt{9-y}\right)dy + 2\int_3^9 \left(\sqrt{9-y}\right)dy$$

$$= 2\left[\int_0^3 (9+y)^{1/2}\,dy - \int_0^3 (9-y)^{1/2}dy + \int_3^9 (9-y)^{1/2}\,dy\right]$$

$$= 2\left[\frac{2}{3}\left[(9+y)^{3/2}\right]_0^3 + \frac{2}{3}\left[(9-y)^{3/2}\right]_0^3 - \frac{2}{3}\left[(9-y)^{3/2}\right]_0^9\right]$$

$$= \frac{4}{3}\left[12\sqrt{12} - 27 + 6\sqrt{6} - 27 - \left(0 - 6\sqrt{6}\right)\right]$$

$$= \frac{4}{3}\left[24\sqrt{3} + 12\sqrt{6} - 54\right]$$

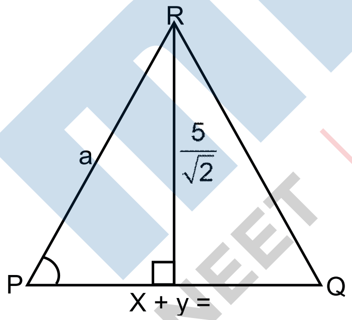
$$= 8\left(4\sqrt{3} + 2\sqrt{6} - 9\right)$$

11. Let R be the point (3, 7) and let P and Q be two points on the line x + y = 5 such that PQR is an equilateral triangle. Then the area of ΔPQR is:

(A) $\frac{25}{4\sqrt{3}}$ (B) $\frac{25\sqrt{3}}{2}$ (C) $\frac{25}{\sqrt{3}}$ (D) $\frac{5}{\sqrt{2}}$

Ans. **(D)**

Sol.



$$\text{Sin } 60° = \frac{5/\sqrt{2}}{a}$$

$$a = \frac{5/\sqrt{2}}{3}$$

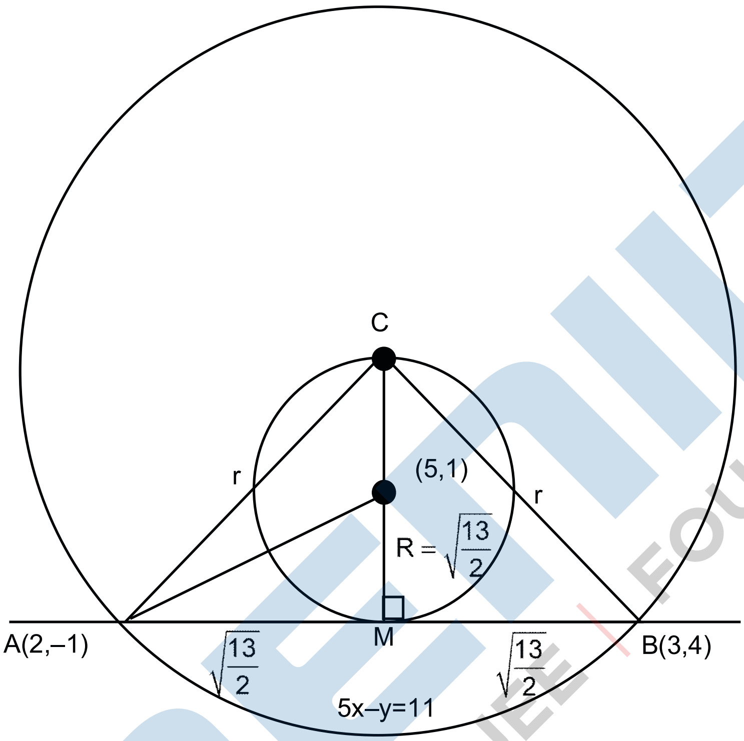
$$\text{Area of } \Delta PQR = \frac{\sqrt{3}}{4}a^2 = \frac{25}{2\sqrt{3}}$$

12. Let C be a circle passing through the points A(2, –1) and B(3, 4). The line segment AB is not a diameter of C. if r is the radius of C and its centre lies on the circle $(x-5)^2 + (y-1)^2 = R = \sqrt{\frac{13}{2}}$, then r^2 is equal to:

(A) 32 (B) $\frac{65}{2}$ (C) $\frac{61}{2}$ (D) 30

Ans. **(B)**

Sol.



$AB = r^2 = \frac{65}{2}$

$r^2 = CM^2 + AM^2$

$= \left(2\times\sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2$

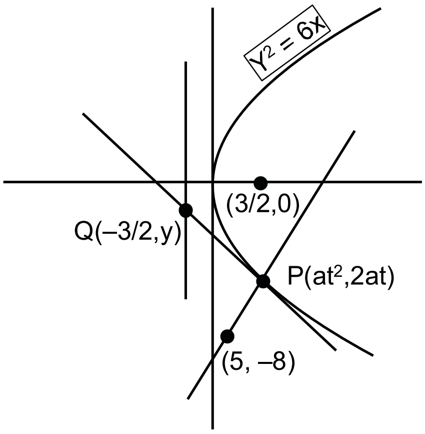
$r^2 = \frac{65}{2}$

13. Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point (5, –8) If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is:

(A) –3 (B) $-\frac{9}{4}$ (C) $-\frac{5}{2}$ (D) –2

Ans. **(B)**

Sol



Equation of normal : $y = -tx + 2at + at^3$ $\left(a = \frac{3}{2}\right)$

since passing through (5, –8), we get t = –2

Co-ordinate of Q: (6, –6)

Equation of tangent at Q : x + 2y + 6 = 0

Put $x = \frac{-3}{2}$ to get $R\,\frac{-3}{2}, \frac{-9}{4}$

14. If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $l_2 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

(A) $\cos^{-1}\left(\frac{29}{4}\right)$ (B) $\sec^{-1}\left(\frac{29}{4}\right)$ (C) $\cos^{-1}\left(\frac{2}{29}\right)$ (D) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Ans. **(B)**

Sol. $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$l_2 : \frac{x-1}{3} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$

$l_3 : \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z+0}{4}$

$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$

angle between l_2 & l_2

$$\cos\theta = \frac{1\times(-3)+(-2)(\alpha/2)+2\times 4}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put α = 3

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right) \text{ t}$$

15. Let the plane 2x + 3y + z + 20 = be rotated through a right about its line of intersection With the plane x– 3y + 5z = 8. If the mirror image of the point $\left(2,-\frac{1}{2},2\right)$ in the rotated plane is B(a, b, c), then:

(A) $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$ (B) $\frac{a}{4}=\frac{b}{5}=\frac{c}{-2}$ (C) $\frac{a}{8}=\frac{b}{-5}=\frac{c}{4}$ (D) $\frac{a}{4}=\frac{b}{5}=\frac{c}{2}$

Ans. **(A)**

Sol. Let equation of rotated plane be:

$(2x + 3y + z + 20) + \lambda(x - 3y + 5z - 8) = 0$

$(2 + \lambda)x + (3 - 3\lambda)y + (1 + 5\lambda)z + 20 - 8\lambda = 0$

Above plane is perpendicular to $2x + 3y + z + 20 = 0$

So, $(2 + \lambda).2 + (3 - 3\lambda).3 + (1 + 5\lambda).1 = 0 \Rightarrow \lambda = 7$

$\Rightarrow$ Equation of rotated plane : $x - 2y + 4z - 4 = 0$

Mirror image of $A\left(2,\frac{-1}{2},2\right)$ in rotated plane is B(a, b, c)

Equation of AB : $\frac{x-2}{1}=\frac{y+1/2}{-2}=\frac{z-2}{4}=k$

Let coordinate of B be $\left(2+k,\frac{-1}{2}-2k,2+4k\right)$

Midpoint of AB is $\left(2+\frac{k}{2},\frac{-1}{2}-k,2+2k\right)$ which will lie on the plane $x - 2y + 4z - 4 = 0$

Hence $k=\frac{-2}{3}$

Therefore B is $\left(\frac{4}{3},\frac{5}{6},\frac{-2}{3}\right)\equiv\left(\frac{8}{6},\frac{5}{6},\frac{-4}{6}\right)$

So, $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$

16. If $\vec{a}\cdot\vec{b}=1, \vec{b}\cdot\vec{c}=2$ and $\vec{c}\cdot\vec{a}=3$, then the value of $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right),\vec{b}\times\left(\vec{c}\times\vec{a}\right),\vec{c}\times\left(\vec{b}\times\vec{a}\right)\right]$ is:

(A) 0 (B) $-6\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$ (C) $12\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)$ (D) $-12\vec{c}\cdot\left(\vec{c}\times\vec{a}\right)$

Ans. **(A)**

Sol. $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}=3\vec{b}-\vec{c}$

$\vec{b}\times\left(\vec{c}\times\vec{a}\right)=\left(\vec{b}.\vec{a}\right)\vec{c}-\left(\vec{b}\cdot\vec{c}\right)\vec{a}=\vec{c}-2\vec{a}$

$\vec{c}\times\left(\vec{b}\times\vec{a}\right)=\left(\vec{c}.\vec{a}\right)\vec{b}-\left(\vec{c}.\vec{b}\right)\vec{a}=3\vec{b}-2\vec{a}$

$\left[3\vec{b}-\vec{c},\vec{c}-2\vec{a},3\vec{b}-2\vec{a}\right]$

$\left(3\vec{b}-\vec{c}\right).\left[\left(\vec{c}-2\vec{a}\right)\times\left(3\vec{b}-2\vec{a}\right)\right]$

$\left(3\vec{b}-\vec{c}\right).\left[3\left(\vec{c}\times\vec{b}\right)-2\left(\vec{c}\times\vec{a}\right)-6\left(\vec{a}\times\vec{b}\right)\right]$

$-6\left[\vec{b}\vec{c}\vec{a}\right]+6\left[\vec{c}\vec{a}\vec{b}\right]$

17. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting almost two heads is :

(A) $\frac{275}{6^5}$ (B) $\frac{36}{5^4}$ (C) $\frac{181}{5^5}$ (D) $\frac{46}{6^4}$

Ans. **D**

Sol. P(H) = x, P(T) = 1 – x

P(4H, 1 T) = P(5H)

${}^5C_1(x)^4\,(1-x)^1 = {}^5C_5\, x^5$

$5(1-x) = x$

$6x = 5 = 0 \qquad x = \frac{5}{6}$

P(atmost 2H)

= P(OH, 5T) + P(1H, 4T) + P(2H, 3T)

$= {}^5C_0\left(\frac{1}{6}\right)^5 + {}^5C_1\frac{5}{6}\cdot\left(\frac{1}{6}\right)^4 + {}^5C_2\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^3$

$= \frac{1}{6^5}(1+25+250) = \frac{276}{6^5}$

$= \frac{46}{6^4}$

18. The mean of the number a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25 M is equal to:

(A) 60 (B) 55 (C) 50 (D) 45

Ans. **(A)**

Sol $\sigma^2 = \frac{\sum_{i=i}^{5}\left(x_1-\bar{x}\right)^2}{n}$

Mean = 6

$\frac{a+b+8+5+10}{5} = 6$

a + b = 7

b = 7 – a

$6.8 = \frac{(a-6)^2+(b+6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$

$34 = (a-6)^2 + (7-a-6)^2 + 4 + 1 + 18$

$a^2 - 7a + 12 = 0 \Rightarrow a = 4$ or $a = 3$

a = 4 a = 3

b = 3 b = 4

$M = \frac{\sum_{i=i}^{5}\left|x_i-x\right|}{n}$

$M = \frac{|a-6|+|b+6|+|8-6|+5-6+|10-6|}{5}$

When a = 3, b = 4

$M = \frac{3+2+2+1+4}{5} \qquad M = \frac{2+3+2+1+7}{5}$

$M = \frac{12}{5} \qquad M = \frac{12}{5}$

$25M = 25\times\frac{12}{5} = 60$

19. Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$, $x \in [-1, 1$. If [a, b] is the range of the function then 4a – b is equal to:

(A) 11 (B) $11-\pi$ (C) $11+\pi$ (D) $15-\pi$

Ans. **(B)**

Sol. $f'(x) = \frac{-2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2$

$= -2\left[\frac{1}{\sqrt{1-x^2}} - \frac{2}{1+x^2} - 3x + 1\right]$

f'(x) < 0 $\Rightarrow$ f(x) is a dec. function

f(1) = π + 5

f(–1) = 5π + 5

Range : [a, b] $\equiv$ [π + 5. 5π + 5]

a = $\pi + 5$, = b = 5π + 5 $\Rightarrow$ 4a – ab =11 – π.

20. Let Δ, $\Delta \in \{\wedge, \vee\}$ b such that p ∇ q $\Rightarrow$ ((p ∇ r) is a tautology. Then (p ∇ q) Δ r is logically equivalent to:

(A) (p Δ q) $\vee$ q (B) (p Δ q) $\wedge$ q (C) (p $\wedge$ q) Δ q (D) (p ∇ q) $\wedge$ q

Ans. (A)

Sol. Case – I If $\Delta \equiv \nabla \equiv \wedge$

(p $\wedge$ q) $\rightarrow$ ((p $\wedge$ q) $\wedge$ r)

It can be false if r is false,

so not a tautology

then (p $\wedge$ q) $\rightarrow$ {(p $\wedge$ q)$\wedge$ r}

Not a tautology

(Check p $\rightarrow$ T, q $\rightarrow$ F)

Case- IV if Δ = $\wedge, \vee$

(p $\wedge$ q) $\rightarrow$ {(p $\wedge$ q) $\vee$r}

Not a tautology

SECTION-B

1. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$ is_______.

Ans. (36)

Sol. $x^4 - 3x - 2x^2 + 3x + 1 = 10$

x = 0 is not the root of this equation so divide it by x^2

$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2 + 2 - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

$$\left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{X}\right) = 0$$

$x - \frac{1}{x} = 0,$ $\qquad$ $x - \frac{1}{x} = 3$

$X^2 - 1 = 0,$ $\qquad$ $x^2 - 3x - 1 = 0$

$X = \pm 1$ $\qquad$ $\gamma + \delta = 3$

$\alpha = 1, \beta = -1$ $\qquad$ $\gamma\delta = -1$

$\alpha^3 + \beta + \gamma^3 + \delta^3$

$1 - 1 + (\gamma + \delta)((\gamma + \delta)^2 - 3\gamma\delta)$

$0 + 3(9 - 3(-1))$

$+ 3(12) = 36$

2. There are ten boys B_1, B_2, …….B_{10} and five girls G_1, G_2, ….. G_5 in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is _________.

Ans. 1120

Sol. n(B) = 10

n(a) = 5

The number of ways of forming a group of 3 girls of 3 boys.

$= {}^{10}C_3 \times {}^{5}C_3$

$$= \frac{10 \times 9 \times 8}{3 \times 2} = \frac{5 \times 4}{2} = 1200$$

The number of ways when two particular boys B_1 of B_2 be the member of group together
$= {}^8C_1 \times {}^5C_3 = 8 \times 10 = 80$
Number of ways when boy B_1 of B_2 hot in the same group together
$= 1200 \times 80 = 1120$

3. Le the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to QO and 6, respectively. If e and ℓ respectively denotes the eccentricity and the lengths of the latus rectum of this ellipse, then $\frac{\ell}{e^2}$ is equal to ___________.

Ans. 4

Sol. $x^2 + y^2 = \frac{9}{4}$ $\qquad$ $y = 4x$

Equation tangent in slope form

$$y = mx \pm \frac{3}{2}\sqrt{(1+m^2)} \quad .....(1)$$

$$y = mx + \frac{1}{m} \quad ......(2)$$

Compare (1) & (2)

$$\pm\frac{3}{2}\sqrt{(1+m^2)} = \frac{1}{m^2}$$

$9m^2 (1 + m^2) = 4$
$9m^4 + 9m^2 - 4 = 0$
$9m^4 + 12m^2 - 3m^2 - 4 = 0$
$3m^2(3m^2 + 4) - (3m^2 + 4) = 0$

$$m^2 = -\frac{4}{3}(\text{Re jected})$$

$$m^2 = \frac{1}{3} \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

Equation of common tangent

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$$

on X axis y = 0
OQ = –3
b = |OQ| = 3
a = 6

$$b^2 = a^2(1-e^2) \Rightarrow e^2 = 1 - \frac{9}{36} = \frac{3}{4}$$

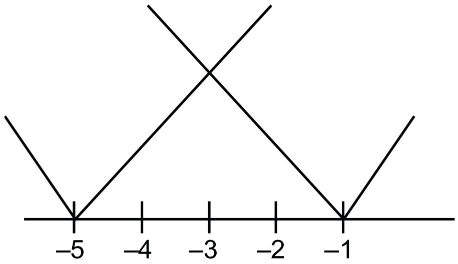
$$e = \frac{2b^2}{a} = \frac{2 \times 9}{6} = 3$$

$$\frac{e}{e^2} = \frac{3}{3/4} = 4$$

4. Let $f(x) = \max\{|x + 1|, |x + 2|, ..., |x + 5|\}$. Then $\int_{-6}^{0} f(x)\,dx$ is equal to ____________.

Ans. 21

Sol. $f(x) = \max\{|x + 1|, |x + 2|, |x + 3|, |x + 4|, |x + 5|\}$



$$\int_{-6}^{0} f(x)\,dx = \int_{-6}^{-3} |x+1|\,dx + \int_{-3}^{0} |x+5|\,dx$$

$$= \int_{-6}^{-3} (x+1)\,dx + \int_{-3}^{0} (x+5)\,dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$$

$$= \left[\left(\frac{9}{2} - 3\right) - (18 - 6)\right] + \left[0 - \left(\frac{9}{2} - 15\right)\right]$$

$$= \left[\frac{3}{2} - 12\right] + \frac{21}{2} = \frac{21}{2} + \frac{21}{2} = 21$$

5. Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the through the origin. Then $y(2)$ is equal to _________.

Ans. **(12)**

Sol. $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx$

$$(x^2 + 4)\frac{dy}{dx} = 2x^3 + 6xy + 8x$$

$$(x^2 + 4)\frac{dy}{dx} - 6xy = 2x^3 + 8x$$

$$\frac{dy}{dx} - \frac{6x}{x^2+4}y = \frac{2x^3+8x}{x^2+y}$$

L.I. $\frac{dy}{dx} + py = \phi$

$$p = \frac{-6x}{x^2+4} \qquad \phi = \frac{2x^3+8x}{x^2+4}$$

$$\text{I.F.} = e\ e^{-\int \frac{6x}{x^2+4}} = e^{-3\log_e(x^2+4)}$$

$$= e^{-3\log_e(x^2+4)} = \frac{1}{(x^2+4)^3}$$

Sol.

$$y.\frac{1}{(x^2+4)^3} = \int \frac{2x^3+8x}{(x^2+4)^3(x^2+4)}dx$$

$$\frac{y}{(x^2+4)^3} = \int \frac{2x(x^2+4)}{(x^2+4)^3(x^2+4)}dx$$

$x^2 + 4 = t$

$2xdx = dt$

$$\frac{y}{(x^2+4)^3}=\int\frac{dt}{t^3}$$

$$\frac{y}{(x^2+4)^3}=\frac{-1}{2(x^2+4)^2}+C$$

Passes through origin (0, 0)

$$0=\frac{-1}{2\times16}+C$$

$$\frac{y}{(x^2+4)^3}=\frac{-1}{2(x^2+4)^2}+\frac{1}{32}$$

$$y=\frac{-(x^2+4)}{2}+\frac{(x^2+4)^3}{32}$$

$$y(2)=-\frac{8}{2}+\frac{8\times8\times8}{32}=12$$

6. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°)=\alpha-\frac{1}{16}\sin(10°)$, then $16+\alpha^{-1}$ is equal to ________.

Ans. **(80)**

Sol. $\sin10°\left(\frac{1}{2}.2\sin20°\sin40°\right).\sin10°\sin(60°-10°)\sin(60°+10°)$

$\left(\sin°\frac{1}{2}\cos20°-\cos60°\right).\frac{1}{4}\sin30°$

$\frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\sin10°\left(\cos20°-\frac{1}{2}\right)$

$\frac{1}{32}(\sin10°-\sin20°-\sin10°)$

$\frac{1}{32}(\sin30°-\sin10°-\sin10°)$

$\frac{1}{32}\left(\frac{1}{2}-2\sin10°\right)$

$\frac{1}{64}(1-4\sin10°)$

$\frac{1}{64}-\frac{1}{16}\sin10°$

7. Let $A=\{n\in N: \text{H.C.F}(n,45)=1\}$ and
Let $B=\{2k : K\in\{1,2,\ldots, 100\}\}$. Then the sum of all the elements of $A\cap B$ is ________.

Ans. **(5264)**

Sol. Sum of elements in $A\cap B$

$$=\underbrace{(2+4+6+\ldots+200)}_{\text{Multiple of 2}}-\underbrace{(6+12+\ldots+198)}_{\text{Multiple of 2,\&3 i.e. 6}}$$
$$-\underbrace{(10+4+6+\ldots+200)}_{\text{Multiple of 5\&2 i.e. 10}}+\underbrace{(30+60+\ldots+180)}_{\text{Multiple of 2,5\&3 i.e. 30}}$$

= 5264

8. The value of the integral $\frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi x^2}{2}-x^3\right)\frac{\sin x}{1+\cos^2 x}dx$ is equal to ________.]

Ans. **(6)**

Sol. $I=\frac{48}{\pi^4}\int_0^{\pi}x^2\left(\frac{3\pi}{2}-x\right)\frac{\sin x}{1+\cos^2 x}dx...(1)$

Apply king property

$I=\frac{48}{\pi^4}\int_0^{\pi}(\pi-x)^2\left(\frac{\pi}{2}+x\right)\frac{\sin x}{1+\cos^2 x}dx...(2)$

(1) + (2)

$I=\frac{12}{\pi^3}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[\pi^2+(\pi-2).x.(\pi-2x)\right]dx...(3)$

Apply king again

(3) + (4)

$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+(\pi-2)(\pi-2x)\right]dx...(5)$

Apply king

$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+(\pi-2)(2x-\pi)\right]dx...(6)$

(5) + (6)

$I=\frac{12}{\pi}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$

Let cosx = t $\Rightarrow$ sinx dx = –dt

$I=\frac{12}{\pi}\int_1^{-1}\frac{-dt}{1+t^2}=6$

9. Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$ and

$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$. Then A + B is equal to ________.

Ans. **(1100)**

Sol. $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$

$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$.

$A=\sum_{i=1}^{10}\min(i,1)+\min(j,2)+...\min(1,10)$

$=\underbrace{(1+1+1+...+1)}_{19\text{ times}}+\underbrace{(2+2+2...+2)}_{17\text{ times}}+\underbrace{(3+3+3...+3)}_{15\text{ times}}$

+ … (1) times

$B=\sum_{j=1}^{10}\max(i,1)+\max(j,2)+...\max(i,10)$

$=\underbrace{(10+10+...+10)}_{19\text{ times}}+\underbrace{(9+9+...+9)}_{17\text{ times}}+...+1\text{1times}$

A + B = 20(1 + 2 + 3 +…+10)

$=20\times\frac{10\times 11}{2}=10\times 110=1100$

10. Let $S=(0,2\pi)-\left\{\frac{\pi}{2},\frac{3\pi}{4},\frac{3\pi}{2},\frac{7\pi}{4}\right\}$. Let y = y(x), $x \in S$, be the solution curve of the differential equation $\frac{dy}{dx}=\frac{1}{1+\sin 2x}, y\left(\frac{\pi}{4}\right)=\frac{1}{2}$. if the sum of abscissas of all the points of intersection of the curve y = (x) with the curve y = $\sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to ________.

Ans. **(42)**

Sol. $\frac{dy}{dx}=\frac{1}{1+\sin 2x}$

$\int dy=\int\frac{dx}{(\sin x+\cos x)^2}$

$\int dy=\int\frac{\sec^2 x}{(1+\tan x)^2}$

$y(x)=\frac{1}{1+\tan x}+C]$

$y\left(\frac{\pi}{4}\right)=\frac{1}{2}=-\frac{1}{2}+C$

C = 1

$y(x)=\frac{-1}{1+\tan x}+1$

$y(x)=\frac{-1+1+\tan x}{1+\tan x}+1$

$y(x)=\frac{\tan x}{1+\tan x}$

Solving with $y=\sqrt{2}\sin x$

$\frac{\tan x}{1+\tan x}=\sqrt{2}\sin x$

$\sin x = 0,\qquad \frac{1}{\sqrt{2}}=\sin x+\cos x$

$x=\pi \qquad \frac{1}{2}=\sin\left(x+\frac{\pi}{4}\right)$

$\sin\frac{\pi}{6}=\sin\left(x+\frac{\pi}{4}\right)$

$x+\frac{\pi}{4}=\pi-\frac{\pi}{6},2\pi+\frac{\pi}{6}$

$x+\frac{5\pi}{6}-\frac{\pi}{4},x=\frac{13\pi}{6}-\frac{\pi}{4}$

$x+\frac{7\pi}{12},x=\frac{23\pi}{12}$

sum of sol.

$=\pi+\frac{7\pi}{12}+\frac{23\pi}{12}$

$=\frac{12\pi+7\pi+23}{12}=\frac{42\pi}{12}=\frac{k\pi}{12}$

$\Rightarrow$ k = 42